

PROBLEMS

6.6.2005

ENTRANCE EXAM FOR ENGINEERING CURRICULUM

The duration of the examination is 2 h 15 min.

Mathematics, logical deduction and physics/chemistry

There are nine problems to solve, each worth 3 points. For each problem, write the answer in the space provided on the answer sheet, and include calculation details whenever the solution requires calculation.

When solving any of problems 7 through 9, you have to choose either the one in physics (A) or the one in chemistry (B). If you solve both, the one with lower score will count.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

The problems are the following:

1.
 - a) Simplify the expression $3a - [3 + 2(a - 1)]$
 - b) Simplify the expression $\frac{2k^2 + 4k}{k + 2}$
 - c) Evaluate the following expression for $a = -5$ and $b = 6$: $\frac{2ab}{3ab + 80} - a^2$

2.
 - a) Solve the following equation for x : $2x - 1 = -(4x - 4)$
 - b) Solve the following equation for μ : $F - \mu mg = ma$
 - c) Solve the following equation for T_1 : $\eta = \frac{T_1 - T_2}{T_1}$

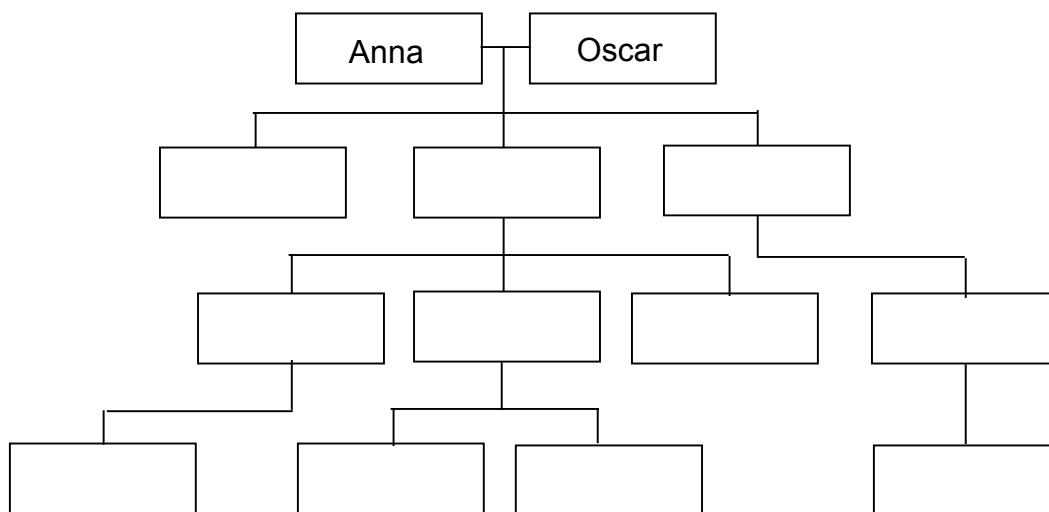
3. After a rise in the price of petrol, the driven mileage dropped by 6.7 %. However, the fuel costs increased by 4.5 %. By what percent did the petrol price rise?

4. A sheet of paper measures 210 mm \times 297 mm. Consider a diagonal from one corner of the sheet to its opposite corner, and choose a point on the diagonal so that its distance to the furthest edge of the sheet is equal to the length of that edge, that is, 210 mm. What is the distance of the point to the nearest edge of the sheet?

5. If you roll two dice, there are $6 \times 6 = 36$ possible outcomes with equal probability. For example, the probability of both dice showing a three is $1/36$, whereas the probability of one dice being a three and the other being a four is $2/36$. Figure out what are the probabilities of the following events: (You can give the answer as a fraction without justifying it.)
 - a) You toss a coin twice, and the first toss is tails whereas the second toss is heads.
 - b) You roll a dice twice, and the sum of the resulting numbers is 4.
 - c) There are two red balls and two blue balls in a box. You draw two balls randomly from the box at the same time, and both of them turn out to be red.

6. Given the following facts, find out the descendants of Anna and Oscar in the family tree below: (Use the family tree below to draft a solution, and then add the names to the copy of the family tree on the corresponding answer sheet.)

- Sally is Harry's daughter.
- Kevin has no children.
- Albert has two brothers, and he is an uncle to three children.
- Lisa has no brothers.
- Vera is an aunt to Eric and his brothers.
- Oliver has one sister but no brothers.
- John's mother is one of Anna's grandchildren.
- Thomas is Peter's grandchild.



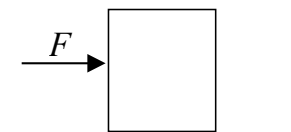
When solving any of problems 7 through 9, you have to choose either the one in physics (A) or the one in chemistry (B). If you solve both, the one with lower score will count.

- 7 A. A 5.0-kg block of ice falls away from the edge of the roof of a block of flats, at a height of 26 m above the ground. Ignoring air drag, find out (a) how long it takes for the ice block to hit the ground, (b) what is the speed at which the ice block hits the ground and (c) how much energy it transfers to the surroundings when it comes to a stop and breaks into pieces. ($g = 9.8 \text{ m/s}^2$)
- 7 B. a) How many grams of gas are there in 0.34 mole of carbon dioxide?
 b) A piece of ice consists of 3.7×10^{23} water molecules. What is the mass of the piece of ice?
 c) How many moles are there in 3.0 kg of calcium chloride (CaCl_2), a salt used, for example, to control dust and ice on roads?

There is a copy of the periodic table of the elements on the last page.
 Avogadro's number is $6.02 \times 10^{23} \text{ 1/mol}$.

- 8 A. This problem includes three parts, marked I, II and III. For each part, write the capital letter of the right alternative in the space provided for the part on the answer sheet. Each right answer gives you one point, provided that you justify the answer correctly. (Include a calculation and a brief explanation if need be.)

- I A heavy, 85-kg crate is lying at rest on a horizontal floor. The coefficient of static friction is 0.30, and that of kinetic friction is 0.24. Find out which of the following statements correctly describes the situation when a horizontal force of 210 N is applied to the crate, as shown in the figure:



- A The crate is at rest and is subject to a 210-N frictional force.
 B The crate is at rest and is subject to a 250-N frictional force.
 C The crate is moving and is subject to a 200-N frictional force.
 ($g = 9.8 \text{ m/s}^2$)

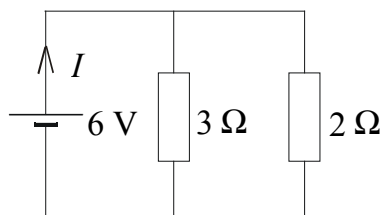
- II There is 0.40 kg of water at 20°C in a vacuum flask. A 0.10-kg piece of ice at 0°C is placed in the flask. Assuming that the heat capacitance of the flask is very small and the heat exchange with the surroundings is negligible, find out which of the following statements correctly describes the situation in the flask after thermal equilibrium has been reached:

- A There is both water and ice at 0°C in the flask.
 B There is 0.50 kg of water at about 0°C in the flask.
 C There is 0.50 kg of water at about 4°C in the flask.

The specific heat of water is $4.19 \text{ kJ}/(\text{kg}^\circ\text{C})$, and the latent heat of fusion of ice is $333 \text{ kJ}/\text{kg}$.

- III The current I in the circuit to the right is

- A 5.0 A
 B 1.2 A
 C 0.83 A



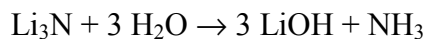
- 8 B. On the answer sheet, there is a table similar to the one below. Fill in the gaps there.

Symbol	Cr		Ag	Cl^-	
Protons		34		17	20
Neutrons		45	62		
Electrons		34			18
Mass number	52			37	40

There is a copy of the periodic table of the elements on the last page.

9 A. The water flow through the turbine of a hydropower plant is $660 \text{ m}^3/\text{min}$, and the vertical height from the tail water level to the intake water level is 14 m. The efficiency of the power plant is 86 %, which means that the power plant converts 86 percent of the change in the potential energy of the water into electrical energy. How much power does the hydropower plant deliver to the grid? ($g = 9.8 \text{ m/s}^2$)

9 B Lithium hydroxide can be produced by the following reaction:



a) How many moles of lithium hydroxide are formed by reacting 7.0 grams of lithium nitride with an excess of water?

b) How many moles of ammonia can be formed if 2.5 moles of lithium nitride is mixed with 4.5 moles of water?

There is a copy of the periodic table of the elements on the last page.

Periodic Table of the Elements

1 H 1.008																	2 He 4.003
3 Li 6.941	4 Be 9.012											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.87	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.69	29 Cu 63.55	30 Zn 65.41	31 Ga 69.72	32 Ge 72.64	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (99)	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
55 Cs 132.9	56 Ba 137.3	57–71 L	72 Hf 178.5	73 Ta 180.9	74 W 183.8	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.6	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89–103 A	104 Rf (261)	105 Db (262)													

L = the lanthanide series (elements 57 to 71)

A = the actinide series (elements 89 to 103)